Sliding Mode Observer for a system of pumping

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***Abstract*-- The sensors can be very expensive and their integration very complex in certain industrial processes. The greatnesses not measured estimated by means of observers are going to allow us to reduce the production cost by avoiding us placing sensors. In the nonlinear case the used observers can be nonlinear sliding mode observers, as the nonlinear sliding mode triangular observer used in our application to reconstruct the state of a system of pumping.**

***Keywords*--Nonlinear systems, system of pumping, observer, sliding mode, sliding mode triangular observer.**

1. INTRODUCTION

The implementation of the laws of command based on the nonlinear model of the system requires the knowledge of the complete state vector of the system at every instant. But, in most of the cases, the only accessible greatnesses of the system are the input and output variables, it is necessary that from this informations to reconstruct the state of the model chosen to elaborate the command. Therefore, the idea bases on the use of an observer [4].

An observer is a dynamic system which we can call it a computer sensor, because it is often implanted on computer to reconstitute or estimate in real time the current state of a system, from the measures and inputs available of the system and knowledge in priori of the model. He allows us then to follow the evolution of the state as information about the system [1].

The possibility of reconstituting internal information on the system by means of the available external greatnesses can be useful for several levels [5]:

_ The command of the process, which requires very often the knowledge of his internal state.

_ The surveillance of the process, through the gaps between the behavior of the observer and that of the process.

For the nonlinear systems, observer synthesis is still an opened problem. One of the classes the most known for the robust observers is the sliding mode observers.

2. OBSERVABILITY AND OBSERVER

The Observability of a process is a very important concept in automatic. Indeed, to reconstruct the state and the output of a system, it is necessary to know, in priori, if the variables of state are observable or not.

Generally, for reasons of technical realization, cost, etc., the dimension of the vector of the output is lower than that of the state. For this reason, at the given moment t, the state x (t) cannot be algebraically deducted from the output y(t) at this moment [4]. On the other hand, under conditions of observability this state can be deducted from the knowledge of inputs and outputs on a past interval of the time.

The purpose of an observer is to supply with a guaranteed precision the current valuation of the state according to inputs and past outputs.

The principal plan of an observer [5] is given in the scheme mentioned in figure 1:

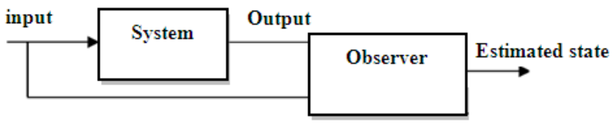


Fig.1.Bloc scheme of an observer.

3. SLIDING MODE OBSERVER

In most part of the problems of command, the complete state is used in the law of command. However in the majority of the cases the state is not completely measurable.

To resolve this problem we use an observer to estimate the complete state of the system.

The synthesis of a sliding mode observer consists to force, by means of discontinuous functions, the dynamics of the errors of estimation of a nonlinear system of order n having p outputs to converge on a variety of order (n - p) said surface of sliding[2].

The attractiveness and the invariance of the surface of sliding are assured by conditions called conditions of sliding [6].

For a nonlinear system, of the shape:

$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \qquad (1)$$

A structure of observer by sliding mode written [5]:

$$\begin{cases} \dot{\hat{x}} = \hat{f}(\hat{x},u) + \Lambda sign(y-\hat{y}) \\ \hat{y} = \hat{h}(\hat{x}) \end{cases} \tag{2}$$

It is a copy of the model, to which we add a corrective term, which assures the convergence of $\hat{x}$ to x . The surface of sliding in that case is given by:

$$s(x) = y - \hat{y} \tag{3}$$

The used term of correction is proportional to the discontinuous function sign applied to the error of output where $sign(x)$ is defined by:

$$sign(x) = \begin{cases} 1 & si \quad x > 0 \\ 0 & si \quad x = 0 \\ -1 & si \quad x < 0 \end{cases} \tag{4}$$

3.1. SLIDING MODE TRIANGULAR OBSERVER

The sliding mode triangular observer was developed for systems which we can put it under the following shape called triangular shape of observation [3]:

$$\begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ . \\ . \\ . \\ \dot{x}_{n-1} \\ \dot{x}_n \end{pmatrix} = \begin{pmatrix} x_2 + g_1(x_1,u) \\ x_3 + g_2(x_1,x_2,u) \\ . \\ . \\ . \\ x_n + g_{n-1}(x_1,x_2,\dots,x_{n-1},u) \\ f_n(x) + g_n(x,u) \end{pmatrix} \\ y = x_1 \end{cases} \tag{5}$$

Where g_i and f_n for i = 1, 2, ..., n are an analytical functions, $x = [x_1\, x_2\, \dots x_n\,]^T \epsilon R^n$ the state of the system, $u \epsilon R^m$ is the vector of input and $y \epsilon R$ the output.

The structure of the proposed observer is the following one [3]:

$$\begin{cases} \begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ . \\ . \\ . \\ \dot{\hat{x}}_{n-1} \\ \dot{\hat{x}}_n \end{pmatrix} = \begin{pmatrix} \hat{x}_2 + g_1(x_1,u) + \lambda_1 sign_1(x_1 - \hat{x}_1) \\ \hat{x}_3 + g_2(x_1,\bar{x}_2,u) + \lambda_2 sign_2(\bar{x}_2 - \hat{x}_2) \\ . \\ . \\ . \\ \hat{x}_n + g_{n-1}(x_1,\bar{x}_2,\dots,\bar{x}_{n-1},u) + \lambda_{n-1} sign_{n-1}(\bar{x}_{n-1} - \hat{x}_{n-1}) \\ f_n(x_1,\bar{x}_2,\dots,\bar{x}_n) + g_n(x_1,\bar{x}_2,\dots,\bar{x}_n,u) + \lambda_n sign_n(\bar{x}_n - \hat{x}_n) \end{pmatrix} \\ \hat{y} = \hat{x}_1 \end{cases} \tag{6}$$

Where the variables $\bar{x}_i$ are given by [3]:

$$\bar{x}_i = \hat{x}_i + \lambda_{i-1} sign_{moy,i-1}(\bar{x}_{i-1} - \hat{x}_{i-1}) \tag{7}$$

With $sign_{moy,i-1}$ indicating the function $sign_{i-1}$ filtered by a low pass filter, the function $sign_i(.)$is put in zero if it exists j ϵ {1,…, *i-1*} such as $\bar{x}_j - \hat{x}_j \neq 0$ (by definition $\bar{x}_1 = x_1$), if not $sign_i(.)$ is taken equal to the classic function $sign(.)$. For this we impose that the corrective term is "active" only if the condition $\bar{x}_j - \hat{x}_j = 0$ for j = 1, 2, …, i-1 is verified.

There is a choice of λ_i Such as the state of the observer $\hat{x}$ converges at a finished time to the state *x* of the system [3]

3.2. STUDY OF THE CONVERGENCE OF THE OBSERVER

Let us consider the dynamic of the error of observation *e* = *x* – $\hat{x}$ and let us proceed step by step [3]-[5].

For $e_1 = x_1 - \hat{x}_1$, we have:

$$\dot{e}_1 = e_2 - \lambda_1 sign(e_1) \tag{8}$$

With: $e_2 = x_2 - \hat{x}_2$

If $\lambda_1 > |e_2|_{max}$ for $t < t_1$ then, the surface of sliding $e_1 = 0$ is attractive and it is reached after finished time t_1 what makes that $\dot{e}_1 = 0$.

There is a continuous function noted $sign_{eq}$ defined by : $e_2 - \lambda_1 sign_{eq}(e_1)$ =0 implying $\bar{x}_2$=x_2 on the surface of sliding.

Because $sign_{eq} = sign_{moy}$ then:

$$\dot{e}_1 = x_2 - (\hat{x}_2 + \lambda_1 sign_{eq}(x_1 - \bar{x}_1)) = x_2 - \bar{x}_2 = 0 \tag{9}$$

Once x_2 known, we pass to the dynamic of e_2.We have after t_1 , $\bar{x}_2$=x_2 what implies that:

$$g_1(x_1,x_2) - g_2(x_1,\bar{x}_2) = 0 \tag{10}$$

Then $\dot{e}_2 = e_3 - \lambda_2 sign(e_2)$, and according to the same reasoning if $\lambda_2 > |e_3|_{max}$ for $t < t_2$, we shall have after a finished time the convergence to the surface $e_1 = e_2 = 0$ and $\dot{e}_2 = 0$ then: $x_3 = \bar{x}_3$ because:

$$\dot{e}_2 = x_3 - (\hat{x}_3 + \lambda_2 sign_{eq}(x_2 - \bar{x}_2)) = x_3 - \bar{x}_3 = 0 \tag{11}$$

By repeating (n –1) time this process, we have after a time t_{n-1} the convergence of all the error of observation to the surface $e_1 = e_2 = \dots = e_{n-1} = 0$ and consequently $\bar{x} = x$, at a finished time t_{n-1} all the state is known and the error of observation is equal to 0.

4. PRINCIPLE OF THE SYSTEM OF PUMPING

The system of pumping studied is composed of a converter type "Buck", of a motor and a centrifugal pump, where the input and the output of the system are respectively, the tension of command and the angular speed [5].

The converter Buck is a supply with cutting which converts a continuous tension in another continuous tension of lower value [5].

The motor will be fed by the output tension of the converter. We consider a motor with direct current with constant magnetic flux and we neglect the magnetic reaction of the armature and the phenomenon of switching [5].

The used pump serves to transmit the kinetic energy of the motor to the fluid [5].

4.1. SYSTEM MODEL IN THE STATE SPACE

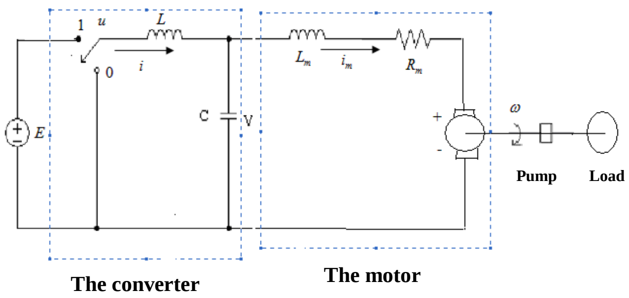


Fig.2. System model of pumping

The electric equations of the converter are:

$$L\frac{di}{dt} = -V + uE \tag{12}$$

And

$$C\frac{dV}{dt} = i - i_m \tag{13}$$

With u designs the cyclic report of the converter which can takes values between 0 and 1.

The electric equation of the motor is:

$$V = R_m i_m + L_m\frac{di_m}{dt} + K_e w \tag{14}$$

With:

$K_e w$: Mains the counter-electromotive force of the motor.

w : Designs the rotation speed of the motor in rad/s.

K_e: Designs the constant of the counter-electromotive force in V/rad/s.

L_m: Designs the inductance of the armature.

R_m : Designs the resistance of the armature.

The centrifugal pump sets against the motor a resisting couple of the shape:

$$C_r = K_r w^2 \tag{15}$$

With: K_r Designs the Coefficient of proportionality in *N.m/(rad /s)*^{2}

The dynamic equation of the electric system motor-pump is:

$$j\frac{dw}{dt} = -B_m w + K_m i_m - K_r w^2 \tag{16}$$

With:

j: Designs the inertia constant of the group motor-pump expressed in *Kg.m*^{2}

B_m: Designs the coefficient of rubbing expressed in (N.m/ rd).

K_m: Designs the constant of the electric couple in (N.m/A).

$K_m i_m$: Designs the electric couple of the motor.

Finally, the mathematical model of the system is the following one:

$$\begin{cases} L\frac{di}{dt} = -V + uE \\ C\frac{dV}{dt} = i - i_m \\ C\frac{dV}{dt} = i - i_m L_m\frac{di_m}{dt} = V - R_m i_m - K_e w \\ j\frac{dw}{dt} = -B_m w + K_m i_m - K_r w^2 \end{cases} \tag{17}$$

If we rewrite this system so as to make appear the vector of state, we obtain the shape given by the equation (16) where we pose $x_1 = w$ as an output of the system.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f(x_1, x_2, x_3, x_4) + gu \end{cases} \tag{18}$$

With:

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = -(&a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \\ &+ a_{11}{x_1}^2 + a_{22}{x_2}^2 + a_{12}x_1x_2 \\ &+ a_{13}x_1x_3 + a_{14}x_1x_4 + a_{23}x_2x_3) \end{aligned}$$

And $g = bE$

The coefficients a_{ij} are deduced and calculated from the following equations:

$$a_1 = \frac{1}{jCLL_m}(K_eK_m + R_m B_m) \tag{19}$$

$$a_2 = \frac{B_m}{Cj}\left(\frac{1}{L_m} + \frac{1}{L}\right) + \frac{R_m}{CLL_m} \tag{20}$$

$$a_3 = \frac{1}{jL_m}(K_eK_m + R_m B_m) + \frac{1}{C}\left(\frac{1}{L_m} + \frac{1}{L}\right) \tag{21}$$

$$a_4 = \frac{B_m}{j} + \frac{R_m}{L_m} \tag{22}$$

$$a_{11} = \frac{R_mK_r}{jCLL_m} \tag{23}$$

$$a_{12} = \frac{2K_r}{Cj}\left(\frac{1}{L_m} + \frac{1}{L}\right) \tag{24}$$

$$a_{13} = \frac{2R_mK_r}{jL_m} \tag{25}$$

$$a_{14} = \frac{2K_r}{j} \tag{26}$$

$$a_{22} = \frac{2R_mK_r}{jL_m} \tag{27}$$

$$a_{23} = \frac{6K_r}{j} \tag{28}$$

$$b = \frac{K_m}{jCLL_m} \tag{29}$$

The nominal parameters of the model are the following ones [5]:

A motor of type: DC motor direct drive brushed type: 828500, of diameter 42mm, and of power:

$P = 42W, V_n = 12V, i_n = 4.25A, w_n = 324.47rad/s$
$L_m = 0.0275\text{H}, R_m = 0.841\Omega, \text{E} = 24\text{V}$

$\text{j} = 0.14.10^{-4}(\text{Kgm}^2)$, $\quad \text{K}_\text{e} = 0.0275(\text{V/rad/s})$,
$K_m = 2.541(N.m/\text{A})$, $B_m = 4.499.10^{-5}(N.m/\text{rad})$.

Static converter: $L = 75mH,\ C = 70mF$

Centrifugal pump: $P_U = 3.2KW,\ w_n = 324.47\text{rad/s}$
$\text{K}_\text{r} = 9.5113.10^{-7}\text{N.m/(rad/s)}^2$

4.2. CONSTRUCTION OF THE SLIDING MODE OBSERVER

For the model of the system studied the sliding mode observer which makes the observed errors converge to zero at finished time can be written as follows:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \lambda_1 sign_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 + \lambda_2 sign_2(\bar{x}_2 - \hat{x}_2) \\ \dot{\hat{x}}_3 = \hat{x}_4 + \lambda_3 sign_3(\bar{x}_3 - \hat{x}_3) \\ \dot{\hat{x}}_4 = f(x_1, \bar{x}_2, \bar{x}_3, \bar{x}_4) + gu + \lambda_4 sign(\bar{x}_4 - \hat{x}_4) \end{cases} \tag{30}$$

With: $\bar{x}_1 = x_1$

$$\bar{x}_i = \hat{x}_i + \lambda_{i-1} sign_{moy}(\bar{x}_{i-1} - \hat{x}_{i-1}) \tag{31}$$

With: $i = 2,3,4$.

$sign_{moy}$ designs the continuous approximation of the function sign as for example:

$$sign_{moy}(\bar{x}_{i-1} - \hat{x}_{i-1}) = arctg\left[\frac{2}{\pi}(\bar{x}_{i-1} - \hat{x}_{i-1})\right] \tag{32}$$

4.3. THE OBSERVER CONVERGENCE

The dynamic of the error of observation is:

$$\begin{cases} \dot{e}_1 = e_2 - \lambda_1 sign(x_1 - \hat{x}_1) \\ \dot{e}_2 = e_3 - \lambda_2 sign(x_2 - \hat{x}_2) \\ \dot{e}_3 = e_4 - \lambda_3 sign(x_3 - \hat{x}_3) \\ \dot{e}_4 = f(x_1, x_2, x_3, x_4) - f(x_1, \bar{x}_2, \bar{x}_3, \bar{x}_4) - \lambda_4 sign(\bar{x}_4 - \hat{x}_4) \end{cases} \tag{33}$$

By the choice of $\lambda_1 > |\text{e}_2|_{max}$ we obtain the convergence of e_1 to zero after the time t_1.

After the time t_1, the state reaches the surface of sliding and on this surface we have: $e_1 = \dot{e}_1 = 0$ Then we have $\bar{x}_2 = x_2$.

In the same way:

By the choice of $\lambda_2 > |\text{e}_3|_{max}$ we obtain the convergence of e_2 to zero after the time t_2.

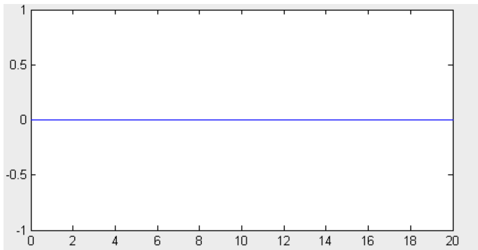
By the choice of $\lambda_3 > |\text{e}_4|_{max}$ we obtain the convergence of e_3 to zero after the time t_3.

By the choice of $\lambda_4 > 0$ we obtain the convergence of e_4 to zero after the time t_4.

4.4. SIMULATION RESULTS:

The simulation under matlab/ Simulink gave the following results:

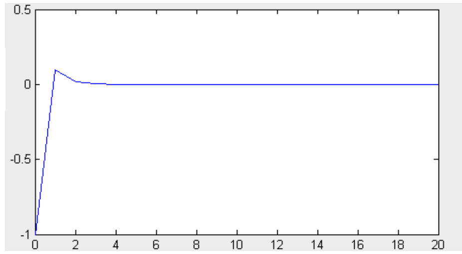
e1 (t)



Time (s)

Fig.3.a. Observation error e_1 (t)

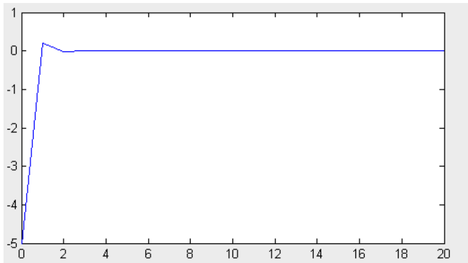
e2 (t)



Time (s)

Fig.3.b. Observation error e_2 (t)

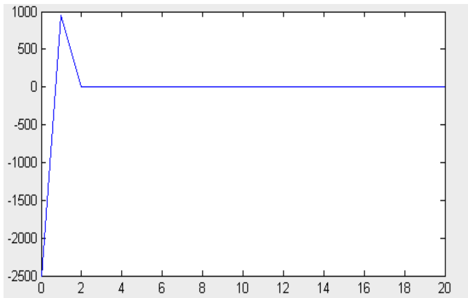
e3 (t)



Time (s)

Fig.3.c. Observation error e_3 (t)

e4 (t)



Time (s)

Fig.3.d. Observation error e_4

Fig.3. Observation errors

On the figure 3, we can visualize the greatnesses of the errors which converge all to zero.

The results of the simulation can inform us on times $t_1 = 0$, $t_2 = 2s$, $t_3 = 2.4s$, $t_4 = 2.8s$ from which the observer converges. The maximum of these times corresponds to the necessary time of convergence of the system.

These results obtained show also the efficiency and the robustness of the sliding mode triangular observer applied to the pumping system.

5. GENERAL CONCLUSION

In this work, we are interested to the realization of a sliding mode triangular observer to determine the dynamic of the errors of observation for a system of pumping.

The results of simulation show the efficiency and the robustness of such observer.

We can add to these advantages the cost that we can win for systems where the sensors are very expensive by report to the integration of a sliding mode observer.

The sliding mode observer is not applicable only on the triangular systems because we can apply it also for the other types of systems and that will make the objective of our next works.

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